PILOT-IN-THE-LOOP PROBLEM AND ITS SOLUTION

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Abstract: *Main purpose of the author is to summarize theoretical backgrounds dealing with mathematical modeling of the human pilot behavior, and to give some approximated models applying Padé approximation method. Importance of this paper is in derivation and application of higher order Padé approximants to model human pilot behavior. This new approach allows to model pilot behavior more precisely than before with applying its first order approximants. The lower and higher model approximants will be analyzed both in time and in frequency domain. The paper deals with derivation of the critical parameters of the human pilot destabilizing the closed loop automatic flight control systems. A new MATLAB® embedded code is generated to analyze the pilot mathematical models, and for both open and closed loop automatic flight control system's analysis.*

Keywords: *human pilot behavior, time delay, critical parameters.*

1. INTRODUCTION & LITERATURE OVERVIEW

Early pioneers of mathematical modeling of the pilots' behavior were McRuer and Krendel. This NATO-report deals with mathematical modeling of human pilots, with analysis of the pilot's behavior in SISO[1] and MIMO[2] automatic flight control systems. In [1] mathematical model of the human pilots depends also on the signals feature to be followed by the pilot. Authors introduced term of the so-called *paper pilot*, which means creation of mathematical model of the pilot as the control element of the automatic flight control systems and widely applied in flight control systems' analysis and preliminary design [1]. Mathematical handbook of G. A. Korn and T. M. Korn is cited as main source for mathematical backgrounds of the problems of approximating time delay [2]. In [3] D. McLean deals with conventional and modern mathematical modeling of the human pilot behavior making difference between aircraft and helicopter pilots. In this textbook time delay of human pilot is approximated using first order Padé approximation, which is in many case may be unsatisfactory and time delay may be approximated by higher order of Padé-approximants. In [4] R. C. Dorf and R. H. Bishop derived mathematical model of the human operator, which has more extended applicability. In that means human operator models can be applied for any kind of drivers (e.g. car, motorcycle, ship, train, ground and air robots etc. drivers and operators). Obviously, the only common thing these models are coinciding is the structure of the mathematical models, while its parameters are quiet different.

Author leans on his scientific papers [5,6,7,8] published before, which are dealing with conventional and modern mathematical methods applied to model human pilot behavior [5], with derivation critical parameters of the human pilot acting in the closed loop automatic flight control system [6, 8], and, with derivation of the complex set of critical parameters of the human pilot in the

[1] Single Input – Single Output
[2] Multi Input – Multi Output

aircraft lateral motion automatic flight control systems [7]

2. PADÉ APPROXIMATION OF THE TIME DELAY

Let us consider the system given in Figure 1 [5,6,7,8]. The transfer function $G(s)$ represents the dynamical system consisting of pure time delay of τ, and transfer function of $G_0(s)$, which is strictly proper and stable. The problem of approximation of the time delay can be formulated as follows: approximate original transfer function of $G(s)=e^{-s\tau}G_0(s)$ by transfer function of $\hat{G}(s)=P_d(s)G_0(s)$, where $P_d(s)=N_d(s)/D_d(s)$ is a rational approximation of time delay of τ. In other words, we want to find $P_d(s)$ so that the closed loop behavior of $\hat{G}(s)$ matches input-output behavior of the original system, of $G(s)$.

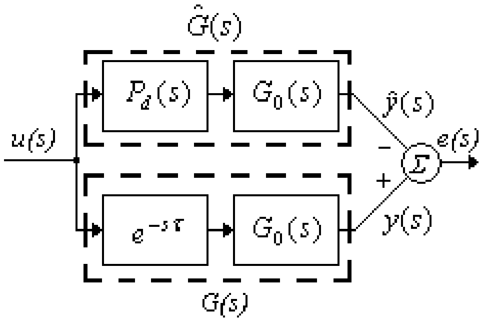

Fig. 1 Block Diagram of the Model Matching Error Problem Formulation

To measure the mismatch we will apply the same input $u(s)$ to both transfer functions of $G(s)$, and $\hat{G}(s)$. By comparing output signals of $y(s)$ and $\hat{y}(s)$ one can derive how $\hat{G}(s)$ approximates $G(s)$, or, how $P_d(s)$ approximates time delay of $e^{-s\tau}$. In control theory, this problem formulated as *model-matching problem.*

The so-called model-matching error (MME) can be given using following equation:

$$MME \hat{=} \sup_{u\neq 0}\frac{\| y-\hat{y}\|_2}{\| u\|_2} \qquad (1)$$

In eq (1) $\| y-\hat{y}\|_2$ denotes the energy of the output error $e=y-\hat{y}$ due to an input signal energy of $\| u\|_2$. The largest possible ratio of the output error energy over the input energy is defined to be *model-matching error*. It is well-known from control theory that model matching error can be found using following formula:

$$MME \equiv MME_{H_\infty} \equiv MME_{L_\infty} \qquad (2)$$

where

$$MME_{H_\infty} = \| G-\hat{G}\|_{H_\infty} \qquad (3)$$

$$MME_{L_\infty} = \sup_{\omega}\left|G(j\omega)-\hat{G}(j\omega)\right| = \sup_{\omega}\left\|G_0(j\omega)\right|\cdot\left| e^{-j\omega\tau}-P_d(j\omega)\right\| \qquad (4)$$

It is obvious, that if MME_{L_∞} is small, than difference between the Nyquist plots of the transfer functions of $G(s)$ and $\hat{G}(s)$ is small. This observation is valid if and only if $G_0(s)$ is stable. Therefore, for the given transfer function of $G_0(s)$ we want to find a rational approximation of $P_d(s)$ for time delay derived by $e^{-s\tau}$ so that the approximation error, or in other words, the model-matching error MME_{L_∞} is smaller than a pre-defined tolerance, say $\delta > 0$.

For further discussion for Padé approximation we will use the following formula [5,6]:

$$e^{-s\tau} \cong P_d(s)=\frac{N_d(s)}{D_d(s)}=\frac{\sum_{k=0}^{n}(-1)^k c_k\tau^k s^k}{\sum_{k=0}^{n}c_k\tau^k s^k} \qquad (5)$$

where coefficients of eq (5) are defined as follows:

$$c_k = \frac{(2n-k)!\cdot n!}{2n!\cdot k!\cdot (n-k)!} \qquad (6)$$

$$n=1,2,3,4,\cdots;\quad k=0,1,2,3,\cdots,n$$

Coefficients of the Padé-approximant for $n\leq 10$ can be found in Appendix 1.

3. MATHEMATICAL MODELS OF THE HUMAN PILOT BEHAVIOR

The simplest mathematical model of the human operator – supposing single reference signal tracking activity – can be derived using Fig. 2 [3,4,5]:

$$Y_p(s) = \frac{x_{out}(s)}{x_{in}(s)} = K_p e^{-s\tau} \qquad (7)$$

where x_{in} is the input signal to be tracked by the pilot, x_{out} is response signal from the pilot, K_p is pilot gain, and finally, τ is time delay of the pilot.

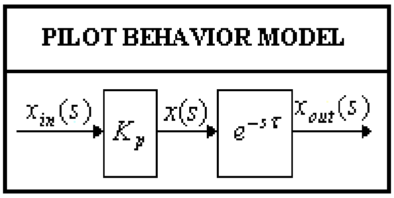

Fig. 2 Mathematical Model of the Pilot Behavior

From eq (7) it is easily can be seen that the human operator behaves as proportional (P) term with pure time delay (TD) [5,6,7,8]. For simplicity let us denote eq (7) for *P-TD*-model. More complicated mathematical model of the human operator – including ability of the pilot to predict events and signals – can be derived using Fig. 3:

$$Y_p(s) = \frac{x_{out}(s)}{x_{in}(s)} = K_p(1+sT_p)e^{-s\tau} \qquad (8)$$

where T_p is the prediction time constant.

From eq (8) it is easily can be derived that the human operator behaves as a proportional-differential (PD) term with pure time delay (TD) [5,6,7,8]. For simplicity let us denote mathematical model of eq (8) as *PD-TD*-model.

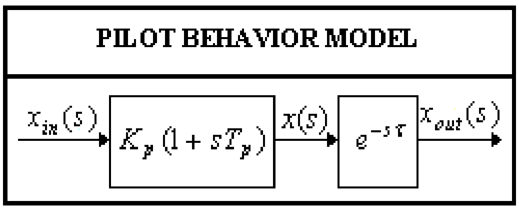

Fig. 3 Mathematical Model of the Pilot Behavior

For further analysis let us consider dynamic model of the muscular acting system of the human operator. Block diagram of the human operator in this particular case can be seen in Fig. 4.

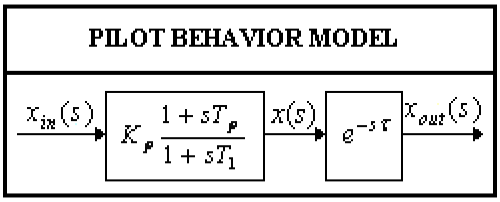

Fig. 4 Mathematical Model of the Human Pilot Behavior

Using Fig. 4 transfer function of the pilot can be derived as it given below:

$$Y_p(s) = \frac{x_{out}(s)}{x_{in}(s)} = K_p \frac{1+sT_p}{1+sT_1} e^{-s\tau} \qquad (9)$$

where T_1 is time constant of the muscular system of the pilot. From eq (9) it easily can be derived that mathematical model of the human operator is proportional-differential (*PD*) first order (1O) term having pure time delay (*TD*) [5,6,7,8]. For further discussions let us denote eq (9) as *PD-1O-TD*-model.

Using Fig. 4 following equation can be derived:

$$Y_p(s) = \frac{x_{out}(s)}{x_{in}(s)} = \frac{x(s)}{x_{in}(s)} \frac{x_{out}(s)}{x(s)} = K_p \frac{1+sT_p}{1+sT_1} e^{-s\tau} \qquad (10)$$

Using eq (10) the following formula can be derived:

$$x(s) = K_p \frac{1+sT_p}{1+sT_1} x_{in}(s) \qquad (11)$$

Input signal $x(t)$ of time delay term of τ can be found using following formula:

$$\dot{x} = -\frac{1}{x} + \frac{K_p}{T_1} x_{in} + \frac{K_p T_p}{T_1} \dot{x}_{in} \qquad (12)$$

For approximation of time delay of τ in eqs (9)-(12) we will use first order Padé approximants.

One can write that

$$Y_p(s)=\frac{x_{out}(s)}{x_{in}(s)}=K_p\frac{1+sT_p}{1+sT_1}e^{-s\tau} \cong K_p\frac{1+sT_p}{1+sT_1}\frac{1-\tau/2}{1+\tau/2} \qquad (13)$$

Modern mathematical representation of the human operator can be given using its state space representation [5,6,7,8]. During derivation of this dynamical model let us choose the state variables as they are given below:

$$x_1 = x_{out} + x \qquad (14)$$

$$x_2 = x \qquad (15)$$

Using eqs (9)-(15) the state and output equations of the human pilot defined on Fig. 3 can be found as follows [5,6,7,8]:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}+\begin{bmatrix}0\\ -\frac{K_pT_p}{T_1}\end{bmatrix}\dot{x}_{in}= \begin{bmatrix}-\frac{2}{\tau} & \frac{4}{\tau}\\ 0 & -\frac{1}{T_1}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ \frac{K_p}{T_p}\end{bmatrix}x_{in} \qquad (16)$$

$$x_{out}=\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (17)$$

Finally, if to consider neuro-muscular sensing, processing and, actuating system of the human pilot following block diagram can be given [1]:

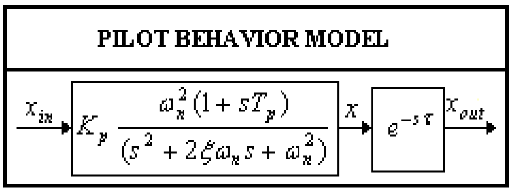

Fig. 5 Mathematical Model of the Human Pilot Behavior

Using Fig. 5 following transfer function of the human pilot can be derived [1,5]:

$$Y_p(s)=\frac{x_{out}(s)}{x_{in}(s)}=\frac{x_{out}(s)}{x(s)}\frac{x(s)}{x_{in}(s)} = K_p\frac{\omega_n^2(1+sT_p)}{(s^2+2\xi\omega_n s+\omega_n^2)}e^{-s\tau} \qquad (18)$$

In eq (18) second order term of

$$\frac{\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2)} \qquad (19)$$

defines mathematical model of the neuromuscular system of the human pilot [1]. It is easy to derive that the second order proportional-differential term of eq (18)

$$Y=\frac{x(s)}{x_{in}(s)}=K_p\frac{\omega_n^2(1+sT_p)}{(s^2+2\xi\omega_n s+\omega_n^2)} \qquad (20)$$

may be rewritten in the following state space model:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\xi\omega_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}x_{in} \qquad (21)$$

$$x=\omega_n^2K_p\begin{bmatrix}1 & T_p\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (22)$$

From equation (18) it easily can be derived that mathematical model of the human pilot is proportional-differential (*PD*) second order (*2O*) term having time delay (*TD*) [5,6,7,8].

For further discussions let us denote eq (18) as *PD-2O-TD*-model.

Let us introduce the following state variable

$$x_3 = x_{out} + x \qquad (23)$$

Time delay τ in eq (18) can de approximated using first order Padé approximants, i.e.:

$$e^{-s\tau}\cong -\frac{s-2/\tau}{s+2/\tau} \qquad (24)$$

Let us substitute eq (24) into eq (18), and convert this mathematical model into the time domain.

After simple mathematical manipulations one can get following state and output equations [1,8, , 0]:

$$\dot{\mathbf{x}}=\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ -\omega_n^2 & -2\xi\omega_n & 0\\ \left(\frac{4}{\tau}K_p\omega_n^2\right) & \left(\frac{4}{\tau}K_pT_p\omega_n^2\right) & \left(-\frac{2}{\tau}\right)\end{bmatrix}\cdot \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 1\\ 0\end{bmatrix}x_{in} \qquad (25)$$

$$x_{out} = \left[-\omega_n^2 K_p \quad -\omega_n^2 K_p T_p \quad 1\right] \left[x_1 \quad x_2 \quad x_3\right]^T \qquad (26)$$

4. TIME DOMAIN ANALYSIS OF THE HUMAN PILOT BEHAVIOR

One of the most important kind of the human pilot activity is the reference signal tracking. Many flight tasks (e.g. semi-automated landing, refueling, air-to-air combat, air-to-ground weapon delivery, terrain following, formation flight, aerobatic close formation flight etc.) are in close relationship with this kind of actuating.

There can be defined some typical input signals to be followed by the pilots, such as step signal, ramp signal, and much other kind of pure or transformed periodical signals (e.g. saw tooth, square signals etc.). In this paper author chose for the time domain analysis the step input function, the ramp input signal, and finally, the square signal [3,4,5].

It is well-known form the previous sections that there are several possible mathematical model of the human pilot to be used during computer simulation. In this paper we will apply dynamical mathematical model of *PD-1O-TD* defined by eq (9), which is represented in Fig. 4. For the computer-aided simulation let us use the following parameters of the mathematical model defined by eq (9):

$$K_p = 10;\ T_p = 1s;\ T_1 = 0{,}4s;\ \tau = 0{,}5s \qquad (27)$$

During computer simulation from the possible set of order of approximation there were chosen the 1st, the 4th, and, the 7th order of approximations. Fig. 6 shows step responses of the human pilot having approximated mathematical model of the time delay. The input signal of the human pilot to be followed by him is $x_{in}(t) = 1(t)$ [9,10].

From Figure 6 it is obvious that increase of order of approximation result in larger amplitudes of the output signal. However, in the time delay zone, increase of the order of the approximation results in oscillations with higher frequencies. It means that error of approximation decreases as its order increases.

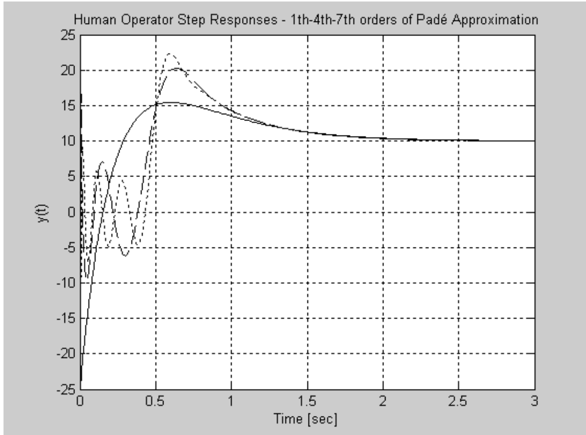

Fig. 6 Step Responses of the Human Operator
'–' 1st '– –' 4th '….' 7th Order Approximation

Fig. 7 shows ramp responses of the human pilot mathematical model. The input signal of the human pilot to be followed by him now is $x_{in}(t) = t$.

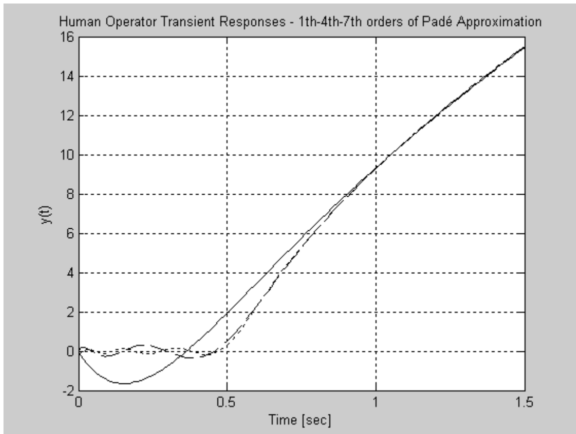

Fig. 7 Ramp Responses of the Human Operator
'–' 1st '– –' 4th '….' 7th Order Approximation

From Fig. 7 it is easily can be seen and derived that increase of the order of approximation results in decrease of the error of the approximation: in the time delay zone magnitude of the output signal $x_{out}(t)$ decreases as order of the approximation is increases while output signal is going to be more and more oscillatory. Finally, let us analyze the human operator behavior when he is tracking the periodical signal. For this kind of analysis author chosen the square signal with frequency of $f = 0{,}3$ Hz, and period time of $T = 1/0{,}3\,\text{sec}$. Results of the computer simulation can be seen in Fig. 8.

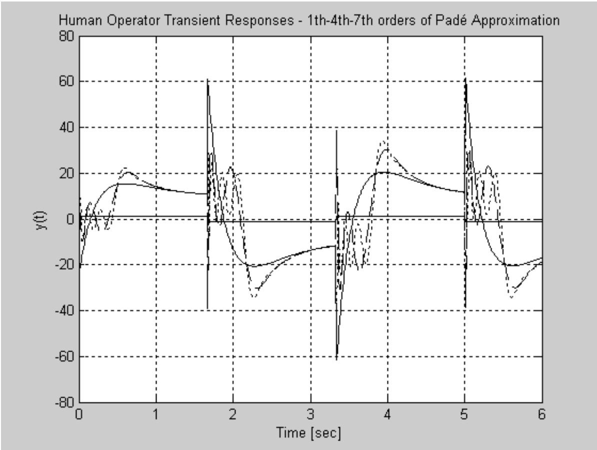

Fig. 8 Transient Responses of the Human Operator '–' 1st '– –' 4th '….' 7th Order Approximation

From Fig. 8 it is easily can be determined that increase of the order of approximation results in less amplitudes in output signal. In time domain of the delay the output signal becomes more oscillatory as order of approximations increases.

5. FREQUENCY DOMAIN ANALYSIS OF THE HUMAN PILOT BEHAVIOR

Typical input signal of the human pilot is the sinusoidal with variable frequencies. Figure 9 shows the response of the human pilot to the harmonic input signal of the sinusoidal with unity gain [9,10]. From Fig. 9 it is obvious that pilot gain for each order of approximation is very close to each other. The phase angle radically decreases as order of approximation is increases [9,10].

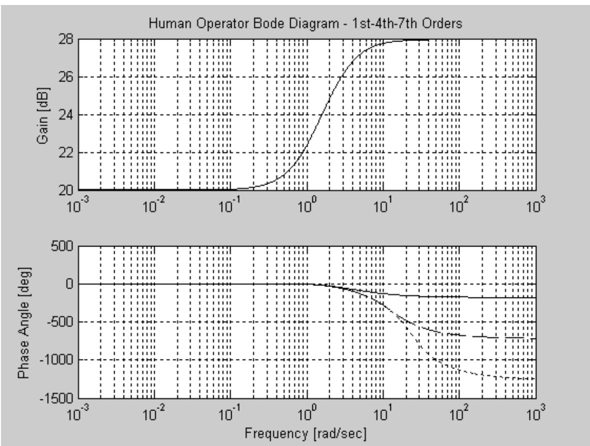

Fig. 9 Bode Diagrams of the Human Operator '–' 1st '– –' 4th '….' 7th Order Approximation

6. COMPARISON OF THE HUMAN PILOT'S BEHAVIOR IN THE TIME DOMAIN

In the practice a question *'what kind of the model of the pilot activity to use for the control system analysis and design?'* often may arise. From theory of automatic flight control systems it is evident that the *pilot-in-the-loop* problem can be characterized with the multi-loop feature, i.e. many flight parameters of such regimes as semi-automated landing of the aircraft airspeed, vertical speed, height of the flight, distance from runway threshold, glide path angle, angular deflection measured from runway centre line etc. must be controlled by the pilot.

From this argue follows that increase value of the flight parameters to be controlled results in decrease of the complexity of the pilot model to be applied during analysis and design of the automatic flight control systems [1,5,8,9,10].

Let us analyze behavior of the human pilot model for several form of its mathematical model supposing second order Padé-approximation for the given time delay. During computer simulation mathematical model defined by eqs (7), (8), (9) and (18). Results of the computer simulation can be seen in Figures 10, 11, and 12.

Fig. 12 represents step responses of the human pilot behavior, when input is step response function of $x_{in}(t) = 1(t)$.

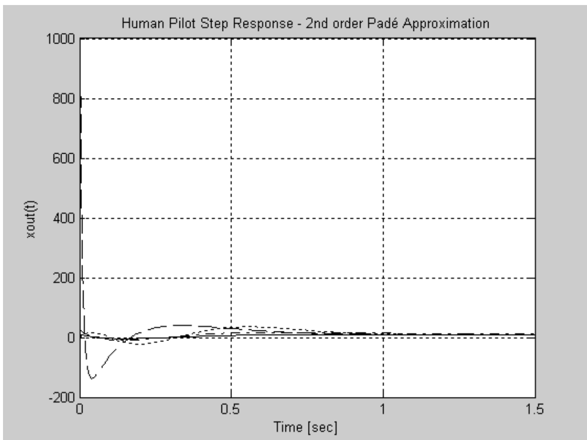

Fig. 10 Step Response of the Human Pilot '–' P-TD '– –' PD-TD '’–.–.' PD-1O-TD '…' PD-2O-TD

Fig. 10 shows ramp responses of different pilot models having input of $x_{in}(t) = t$.

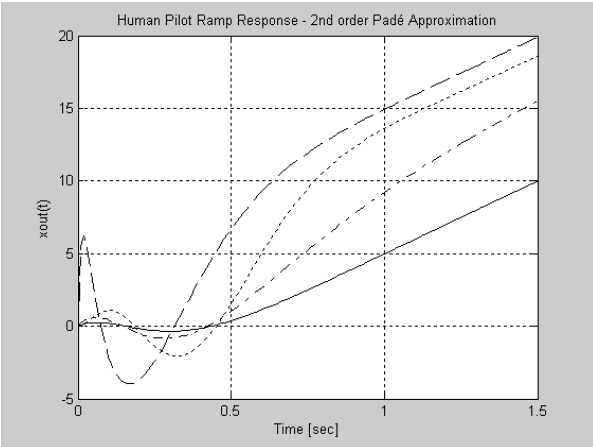

Fig. 11 Ramp Response of the Human Pilot
'–' P-TD '– –' PD-TD ''–.–.' PD-1O-TD '…' PD-2O-TD

Fig. 11 shows transient responses of different human pilot mathematical models induced by square periodical signal with unity gain and frequency of 0,3 Hz.

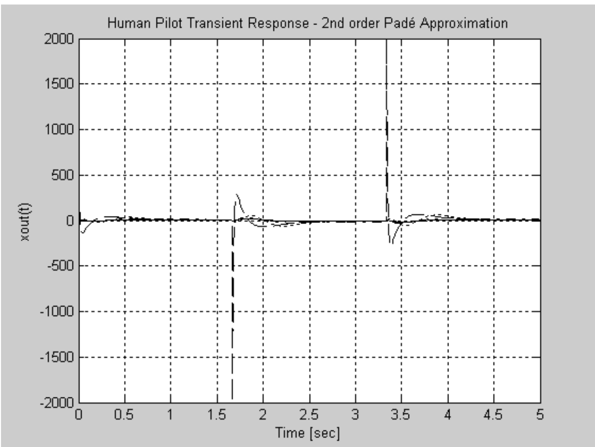

Fig. 12 Transient Response of the Human Pilot
'–' P-TD '– –' PD-TD ''–.–.' PD-1O-TD '…' PD-2O-TD

Figures 10, 11, 12 clearly show that if to add *D*-term to the proportional (see eq (7)) it will result in more oscillatory system (see eq (8)) with large amplitudes (dashed line on the figures). Introducing fist order term to eq (8) will result in damped system reducing oscillatory feature (dash-dot line on the figures). Application of the second order term of eq (18) in comparison with system defined by eq (9) results in more oscillatory behavior (dotted line on the figures).

Using the method given above human pilot model behavior can be compared also for higher orders of the Padé-approximation.

7. DERIVATION OF THE CRITICAL PARAMETERS OF THE PILOT'S ACTIVITY

Knowledge of the human pilot behavior is very important from the flight safety aspects. It is difficult to model a human pilot having mathematical model considering all possible conditions. Even common mathematical models of the human pilot can be applied with great success. Purpose of the author is to show how to determine critical parameters of the human pilot? It is well known that there are many parameters of the pilot (e.g. gain, time delay, time constants, damping rations, natural frequencies etc.) which can be analyzed and also their critical value can be found.

Due to its importance author will deal only with determination of the critical time delay of the human operator yielding instability of the control loops of the automatic flight control systems. Results and proposals of this paper can be applied for extension of the analysis shown is this article. The general method recommended by the author is well known from control systems theory but the paper suggests the new field for its application.

Pilot is the most important element in the aircraft steering system. Even if aircraft has modern control system for maneuvering driven by digital computer pilot must has the right to take control over aircraft and steer it manually. Automation of the aircraft flight phases induced the need to design semi-automated automatic flight control systems, which suggest for the pilot what kind of actuation to carry out. For this purpose high level technology displays are used in the cockpit.

Semi-automated aircraft steering is very useful because pilot takes active part in actuation process and do not reduce his ability. During flight phases semi-automated steering can be applied: semi-automated landing, refueling, air-to-air combat, dog fight, air-to-ground weapon delivery, terrain following, formation flight, aerobatic flight, close formation flight etc.

7.1. Derivation of the Critical Value of Human Pilot Time Delay

During semi-automatic, or manual control of the aircraft one of the problem to be solved by the pilot is reference signal tracking or, following commands suggested by the automatic flight control system, or other systems (e.g. navigation system, radar system, weapon system etc.). As it was said before commands are listed on the display: e.g. turn left, turn right, accelerate, decelerate, descend, climb, etc.

For example, in this paper the single loop automatic flight control system is analyzed. In this particular case pilot has to control only one flight parameter. Let us choose for analysis the roll angle control system. In this system the task of the pilot to track the reference signal of the roll angle $\gamma_R(t)$ indicated on the display.

Block diagram of the semi-automated roll angle control system can be seen in Fig. 13 [3,4,5,8,9,10].

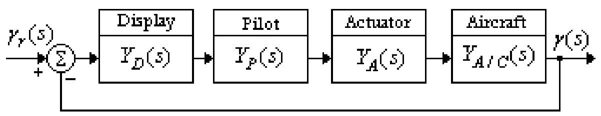

Fig. 13 Pilot-in-the Aircraft Roll Angle Control System

Flight parameters and data are indicated on the displays. It is supposed that display has no time delay and, any time constants. This condition is strongly satisfied for modern analogue and digital displays.

Transfer function of the display can be formulated as follows [3,5]:

$$Y_D(s) \cong 1. \tag{28}$$

Let us take into consideration for the modeling of pilot behavior mathematical model of the human operator. Regarding [5] transfer function, and model parameters are as follows:

$$Y_P = K_p(1+sT_p)\,e^{-\tau s} \cong K_p(1+sT_p)\frac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s} \tag{29}$$

In eq (29) later we will consider following parameters [5,6,7,8]:

$$K_p = 10,\ T_p = 1\,\text{sec}. \tag{30}$$

From eq (29) it is evident that model of the aircraft applied in this section is proportional-differential (*PD*) one with time delay (*TD*), which is approximated with the first order Padé-approximation given in.

Ailerons of the aircraft are deflected using hydraulic actuator.

The simplified mathematical model of the actuator can be defined as it given below:

$$Y_A(s) = \frac{20}{20+s} = \frac{1}{1+0{,}05s} \cong 1 \tag{31}$$

Lateral motion dynamics of the aircraft is supposed to be the so-called single degree of freedom approximation derived by [3,5] and its transfer function is as follows:

$$Y_{A/C}(s) = \frac{\gamma(s)}{\delta_A(s)} = \frac{0{,}21}{s(s+0{,}9)} \tag{32}$$

where $\delta_A(s)$ is the angular deflection of the ailerons, or input of the aircraft, $\gamma(s)$ is roll angle, or in other words, response of the aircraft to its input signal.

From [1] it is evident that pilot model parameters depend upon complexity of the task to be solved by the pilot, and also upon physical and psychical ability of the pilot. Among these parameters time delay is the most important because its presence tends closed loop automatic flight control system to its stable working boundary [3,5,9,10]. Let us derive τ_{crit}, which leads closed loop automatic flight control system to its stable working conditions. For this purpose let find the closed loop automatic flight control system transfer function related to reference input signal of $\gamma_r(s)$.

The closed loop transfer function of the investigated system can be derived using Fig. 3, i.e.:

$$W(s)=\frac{\gamma(s)}{\gamma_R(s)}=\frac{Y_D(s)Y_P(s)Y_A(s)Y_{A/C}(s)}{1+Y_D(s)Y_P(s)Y_A(s)Y_{A/C}(s)}\cong\frac{Y_P(s)Y_{A/C}(s)}{1+Y_P(s)Y_{A/C}(s)} \quad (33)$$

Substituting data defined by eqs (28)-(32) into eq (33) yields to closed loop transfer function formula:

$$W(s)=\frac{0{,}21\cdot(10+10s)(1-\tau/2)}{(s^2+0{,}9s)(1+\tau/2)+0{,}21\cdot(10+10s)(1-\tau/2)} \quad (34)$$

In control theory there are many available methods for determination of the closed loop control system stability. Some of them are graphical, others are algebraic ones. These methods allow deriving stability conditions of the closed loop system. Other possible application of the algebraic stability criteria is finding critical parameter of the closed loop control system [3,4,5,9,10]. Using stability criteria formulated by Hurwitz closed loop control system is stable if and only if

1. all coefficients of the characteristic polynomial are positive ones, say a_i>0. This is the necessary stability condition;
2. algebraic minors on the main diagonal of the Hurwitz-determinant are positive, say Δ_i>0. If there is a single determinant with negative value, the closed loop control system is unstable. If $\Delta_i=0$, the system in upon stable working boundary and this condition can be used for determination of the critical parameters of the control system. This is the sufficient condition of the closed loop stability.

Let us find the characteristic polynomial of the closed loop control system, which is the denominator of the transfer function of eq (6). It is supposed that the only variable parameter is the pilot time delay τ while all other parameters are supposed to be constant. One can easily write that:

$$K(s)=(s^2+9s)(1+\tau/2\, s)+0{,}21\cdot(10+10s)(1-\tau/2\, s)=0 \quad (35)$$

After some simple mathematical procedures we get the following third order characteristic polynomial, i.e.:

$$K(s)=\frac{\tau}{2}s^3+(1-0{,}6\tau)s^2+(3-1{,}05\tau)s+2{,}1=a_o s^3+a_1 s^2+a_2 s+a_3=0 \quad (36)$$

Applying necessary stability conditions using coefficients of eq (36) one can determine following stability inequalities:

$$a_0=\tau/2>0 \;\rightarrow\; >0\text{ s} \quad (37)$$

$$a_1=1-0{,}61\tau>0 \;\rightarrow\; \tau<1{,}6666\text{ s} \quad (38)$$

$$a_2=3-1{,}05\tau>0 \;\rightarrow\; \tau<2{,}8571\text{ s} \quad (39)$$

From eqs (37)-(39) it is obvious that for the stable working closed loop control system the human pilot time delay must lie in the following range:

$$0<\tau<1{,}6666\text{ s}. \quad (40)$$

For the next step let us find sufficient conditions of stability using Hurwitz-determinant.

The Hurwitz-determinant can be found using coefficients of the characteristic polynomial. One can write that:

$$\Delta_3=\begin{vmatrix}1-0{,}6\tau & 2{,}1 & 0\\ \tau/2 & 3-1{,}05\tau & 0\\ 0 & 1-0{,}6\tau & 2{,}1\end{vmatrix} \quad (41)$$

Using eq (41) the following algebraic minors leaning on main diagonal can be found. If we suppose that the system on the boundary of the stable working, following determinants can be derived [7]:

$$\Delta_i=0 \quad (42)$$

From eq (41) we can find the following stability conditions:

$$\Delta_1=1-0{,}6\tau=0 \;\rightarrow\; \tau_{crit}=1{,}6666\text{s} \quad (43)$$

$$\Delta_2=\begin{vmatrix}1-0{,}6\tau & 2{,}1\\ \tau/2 & 3-1{,}05\tau\end{vmatrix}=0 \quad (44)$$

$$0{,}63\tau^2-3{,}9\tau+3=0\rightarrow \begin{matrix}\tau_{1crit}=5{,}2904\text{ s}\\ \tau_{2crit}=0{,}9001\text{ s}\end{matrix} \quad (45)$$

$$\Delta_3=\begin{vmatrix}1-0{,}6\tau & 2{,}1 & 0\\ 0{,}5\tau & 3-1{,}05\tau & 0\\ 0 & 1-0{,}6\tau & 2{,}1\end{vmatrix}=2{,}1\cdot\Delta_2=0 \quad (46)$$

$$\Delta_2 = 0 \rightarrow \begin{matrix} \tau_{1crit} = 5{,}2904\,s \\ \tau_{2crit} = 0{,}9001\,s \end{matrix} \quad (47)$$

From eqs (42)-(47) critical parameter of the human pilot time delay destabilizing closed loop automatic flight control system can be easily derived to be:

$$\tau_{crit} = 0{,}9001\,sec \quad (48)$$

Time delay domain defined from necessary stability conditions and given by eq (40) is limited with time delay defined for the sufficient stability conditions given by eq (48). Stability conditions for the closed loop automatic flight control system given in Figure 13 can be derived as follows:

$$0 < \tau < 0{,}9001\ sec \quad (49)$$

Let us calculate the step response of the closed loop automatic flight control system. In this particular case reference signal of the system to be followed by the human pilot is

$$\gamma_r(t) = 1(t) \quad (50)$$

Let the set of time delays considered during computer simulation be as follows:

$$\tau_{stab} = 0{,}3 sec;\ \tau_{crit} = 0{,}9 sec; \quad \tau_{unstab} = 1 sec \quad (51)$$

Results of the computer simulation can be seen in Fig. 14.

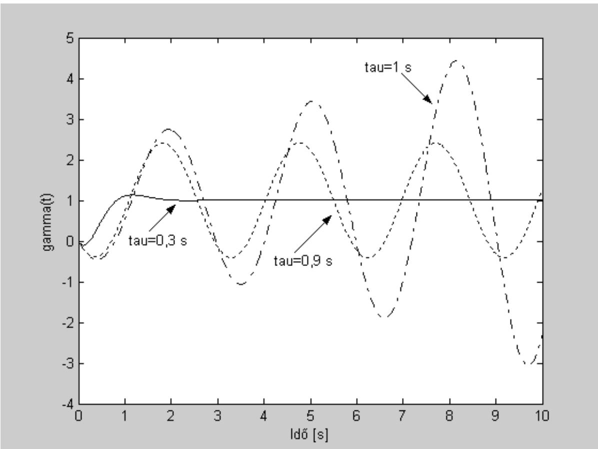

Fig. 14 Closed Loop Automatic Flight Control System Step Responses

Figure 14 shows that for small value of time delay, say $\tau_{stab} = 0{,}3 sec$, the closed loop step response is stable: the roll angle has bounded value. Transient response time is small. It means that pilot is able to track the reference signal with no static error and the closed loop control system is stable.

In case of critical time delay of $\tau_{crit} = 0{,}9 sec$ the closed loop system including human pilot has harmonic, periodical response with constant amplitudes. In other words, pilot unable to track the bounded reference signal.

Finally, having unstable time delay of $\tau_{unstab} = 1 sec$, closed loop automatic flight control system with the pilot inside has unstable response, which is harmonic signal with increasing amplitudes. It is evident that in this particular case pilot loosing the control over the aircraft and may generate the so called *pilot induced oscillation (PIO)* which can be dangerous for flight safety. In worst case situation *PIO* can lead to damage of the airframe and to fatal accident of the aircraft.

Dynamic performances of the closed loop automatic flight control system were found for three different values of the time delay defined by eq (51) and put into Appendix 2.

From the Appendix 2 it is evident that for $\tau_{stab} = 0{,}3 sec$ closed loop automatic flight control system is stable, and has eigen values of $\lambda_1 = -1{,}06$, and $\lambda_{2,3} = -2{,}2 \pm 2{,}89j$ on the left side of the complex plane, which tells about stability.

For critical value of the time delay of $\tau_{crit} = 0{,}9 sec$ closed loop has a pair of complex roots of $\lambda_{2,3} = -1{,}32 \cdot 10^{-4} \pm 2{,}14j$, which is practically lies on the imaginary axis of the complex plane. These roots generate harmonic response of the closed loop automatic flight control system.

In case of $\tau_{unstab} = 1 sec$ closed loop automatic flight control system has a pair of roots on the right side of the complex plane, say, $\lambda_{2,3} = 0{,}109 \pm 2{,}03j$, which generates unstable response from the closed loop automatic flight control system.

8. CONCLUSIONS

Human operators are still one of the most important ‘part’ of the control systems. They may monitor the physical processes, or actively actuate in the control systems. Since operator acts as simple term of the closed loop control system it is necessary to model his activity, and, to take into consideration. Modeling human pilots is important from many aspects of aircraft maintenance both in the air and on the ground. His mathematical model depends upon complexity of the system in which he acts, upon the level of his training, upon his physical and psychical conditions, and finally, depends on signals' characteristics to be followed.

The paper dealt with determination of the human pilot's critical parameters. Author introduced widely applied mathematical models of the human operator. Paper showed a new field of application of the classical Hurwitz stability criteria. A new example was presented how it can be used for purposes of derivation of critical parameters of the pilot.

Note and underline that for complex analysis of critical parameters of the human operator (e.g. gain K_p, and prediction time constant T_p) also must be determined. Conditions and requirements for stability of the closed loop automatic flight control system must be satisfied for all possible parameters of the human operator for all possible aircraft dynamics, i.e. for all possible flight conditions and regimes.

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